Alternative Computation of Dean's Overfill Ratio

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Abstract: An alternative graphical expression is presented for the computation of overfill ratio using the Dean method. The expression yields the same numeric results as the original Dean curves, but allows simple computation of the overfill ratio from a single curve for an unlimited range of sediment cases. The physical significance of results yielded from the Dean method is contrasted with those from the James-Krumbein method. A convenient numeric means to directly translate between phi and millimeter grain sizes is additionally noted.

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Introduction

In beach nourishment or analogous beach disposal projects, the grain size compatibility of dredged ("borrow") sediments and existing ("native") beach sediments is typically expressed by the overfill ratio. The overfill ratio, also known as the overfill factor, describes the volume of borrow sediment that, in theory, will ultimately yield a residual *unit* volume of sediment on the beach after grain sorting and losses. That is, the overfill factor attempts to account for the natural loss of some fraction of the borrow sediment that is finer than the native (or stable existing) beach sediment. For example, an overfill factor of 1.15 suggests that 115 m^3 of borrow sediment must be placed to yield 100 m^3 of residual fill on the beach; or, equivalently, that $1/1.15=87\%$ of the borrow sediment will remain as residual in-place beach fill after sediment sorting losses.

One or both of two methods to compute the overfill ratio are routinely applied: the James-Krumbein method (Krumbein and James 1965; James 1974, 1975) and the Dean method (Dean 1974). Of the two, the latter typically yields less conservative results; i.e., smaller overfill factors. The James-Krumbein (JK) method is also referred to as the "Shore Protection Manual" or "SPM" method (USACE 1984), and assumes that the fraction of borrow sediments that is coarser than the native sediments will be winnowed out from the beach fill analogously as the finer sediments. In contrast, the Dean method assumes that only the finer fraction will be lost; and the method offsets these losses by the fraction of borrow material that is coarser than the native material. The Dean method thus yields less conservative (smaller) values for the overfill ratio.

James (1975) introduced the adjusted SPM method which is intended to partly credit (retain) the fraction of the borrow material that is coarser than the native. Hobson (1977) and Stauble (1986) introduced further modifications to account for the fraction of borrow material less than 0.06 and 0.125 mm, respectively. The adjusted SPM method, including Hobson's and Stauble's modifications, generally yields values of the overfill ratio that fall between or below the JK and Dean methods.

Computation of the overall factor from both methods is based upon the mean grain size diameters of the borrow and native sediments and the sorting parameter (standard deviation) of the borrow sediment. The James-Krumbein (JK) method additionally accounts for the sorting parameter of the native sediment. Using these grain-size data, the overfill factor is then typically determined from a family of curves published for the JK method (James 1974; USACE 1984, 2001) and for the Dean method (Dean 1974; Dean and Dalrymple 2002). The published curves for the Dean method are shown in Fig. 1.

In practice, a disadvantage of the Dean method is that some cases of interest lay beyond the axes of the published curves. These cases include, for example, sediments coarser than 1.0 mm mean grain size (ϕ value <0), fine-grained well-sorted sediments, the Example Problem for overfill ratio calculation in the Coastal Engineering Manual (USACE 2001), among other problems of practical interest. The semilogarithmic axes and curvilinear plots make extrapolation of the overfill factor curves beyond the plotted limits difficult or inaccurate; and interpolation along the axes and between the curves within the plotted limits is not straightforward. Direct numeric computation of the overfill factor from the published equations is fairly complicated and beyond the routine means of typical practitioners.

As presented below, the writer found that Dean's curves (Fig. 1) can be expressed as a single, nonlinear curve by which the overfill factor is directly determined by a single parameter. This parameter is the difference between the borrow and native sediments' mean grain sizes, divided by the sorting parameter of the borrow sediment. The results allow computation of the Dean overfill factor over an unlimited numeric range.

It is noted that the overfill ratio is of principal value in comparing the relative merits of alternative beach fill borrow sources, or the general suitability of fill material for beach placement, as a function of grain size compatibility. Many environmental regulatory agencies require computation of the overfill ratio as one

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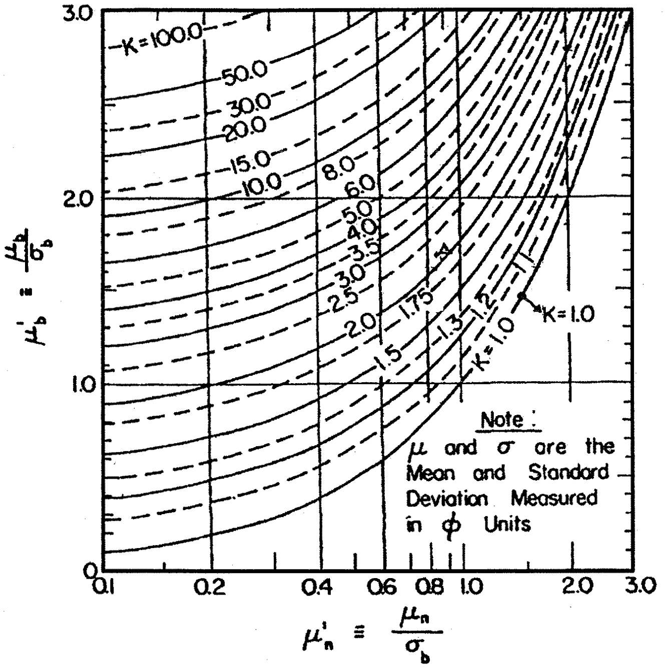

Fig. 1. Dean's overfill ratio curves (Dean 1974, ASCE)

means by which to gauge the suitability of proposed beach fill material. The overfill ratio can be likewise used to estimate the required beach fill volume; though an alternative means to do so is the equilibrium beach profile concept (Dean 1991, 2002). The latter approach contrasts the predicted beach profile shape (determined as a function of grain size) with the native or existing beach profile shape. Unlike the overfill ratio, this method does not presume that the beach has any "memory" of its prior (native) grain size. Rather than a discrete volume multiplier or fill factor, it yields an estimate of the fill volume required to achieve a given width of beach. The method requires that the existing and post-nourishment beach profiles are accurately modeled as a predicted, equilibrium shape, and in its simplest form, as a function of only the mean sediment grain sizes. The purpose of the present Technical Note is not to assess the relative merits or validity of any of these methods. Instead, this Technical Note is intended only to present a useful simplification of the method for computing the original Dean overfill factor.

Method

As introduced, computation of the Dean overfill ratio, or overfill factor, K, is based upon the mean diameter of the borrow (fill) sediment M_b, the mean diameter of the native beach sediment M_n, and the standard deviation (variously referred to as sorting parameter) of the borrow sediment σ_b. Each of these are expressed in phi size units ϕ, where ϕ is the negative base-2 logarithm of the grain size d in millimeters:

$$\phi = -\log_2(d) \tag{1}$$

Eq. (1) can be rewritten by use of the identity

$$\log_a x = (\log_b x)/(\log_b a) = (\log_b x)(\log_a b) \tag{2}$$

where for $a=2$, $b=10$, and $x=d$, Eq. (1) becomes

$$\phi = -3.322\log_{10}(d) \tag{3a}$$

or

$$d = 10^{-0.301\phi} \tag{3b}$$

Eqs. (3a) and (3b) provide a useful means by which to directly convert sediment grain size between millimeters (d) and phi units (ϕ) without recourse to published tables or base-2 functions (the latter being a very rare feature on most practitioners' calculators).

Adopting the formulas in the CEM (USACE 2001), the mean diameter M and standard deviation σ are described as

$$M_b = (\phi_{16} + \phi_{50} + \phi_{84})_b/3 \tag{4a}$$

$$M_n = (\phi_{16} + \phi_{50} + \phi_{84})_n/3 \tag{4b}$$

$$\sigma_b = \frac{(\phi_{84} - \phi_{16})_b}{4} + \frac{(\phi_{95} - \phi_5)_b}{6} \tag{4c}$$

where the subscript b refers to the borrow (or fill) sediment and n refers to the native sediment. The values ϕ_y refer to the grain size, in phi units, for which y percent of the sediment is *coarser*. For example, $\phi_{84b}=2.6$ implies that 84% of the borrow sediment is coarser than 2.6 phi (0.165 mm). By definition, computation of the overfill ratio by either the JK or Dean methods requires that $\phi_{84} > \phi_{16}$, or equivalently, $d_{84} < d_{16}$, such that $\sigma > 0$.

Numerically, Dean's overfill factor K is determined by iterative solution of two coupled equations described by Dean (1974)

$$K[1 + \mathrm{erf}(\phi_* - M'_b)] = 2 \tag{5a}$$

$$K(2\pi)^{-1/2}\exp[-(\phi_* - M'_b)^2/2] \geqslant M'_b - M'_n \tag{5b}$$

where $\mathrm{erf}(x)$=error function defined as

$$\mathrm{erf}(x) = \sqrt{\frac{2}{\pi}}\int_0^x \exp\left(\frac{-t^2}{2}\right)dt \tag{6}$$

where t=dummy variable of integration. The values M'_b and M'_n=dimensionless terms defined by Dean, and which appear on the axes of Fig. 1, as

$$M'_b = M_b/\sigma_b \tag{7}$$

and

$$M'_n = M_n/\sigma_b$$

In combining and solving Eqs. (5a) and (5b) for K, the parameter ϕ_* is computed iteratively for a given pair of values M'_b and M'_n such that

$$2/[1 + \mathrm{erf}(\phi_* - M'_b)] \geqslant (M'_b - M'_n)(2\pi)^{1/2}\exp[(\phi_* - M'_b)^2/2] \tag{8}$$

In this way, for the present work, values of K were computed for a wide range of M'_b and M'_n values to reproduce the results originally published in Dean (1974) with log-normal axes; i.e., as shown in Fig. 1. Subsequently, the results were plotted on *linear* axes, as shown in Fig. 2.

From Fig. 2, it is seen that the overfill ratio curves, K, are of uniform linear (1:1) slope. This allows the curves to be accurately extrapolated beyond the axes' limits—including for negative values (that is, mean grain sizes larger than 1.0 mm).

The curves' unity slope allows that the value of K for some arbitrary pair (M'_n, M'_b) is equal to the value of K for $(0, M'_b - M'_n)$. Visually, this is akin to seeking some point (M'_n, M'_b) in

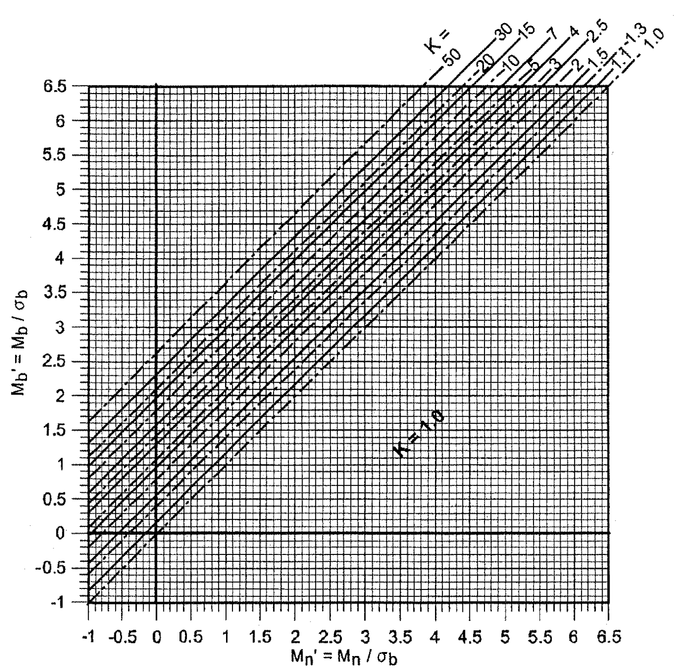

Fig. 2. Dean's overfill ratio curves plotted on linear axes

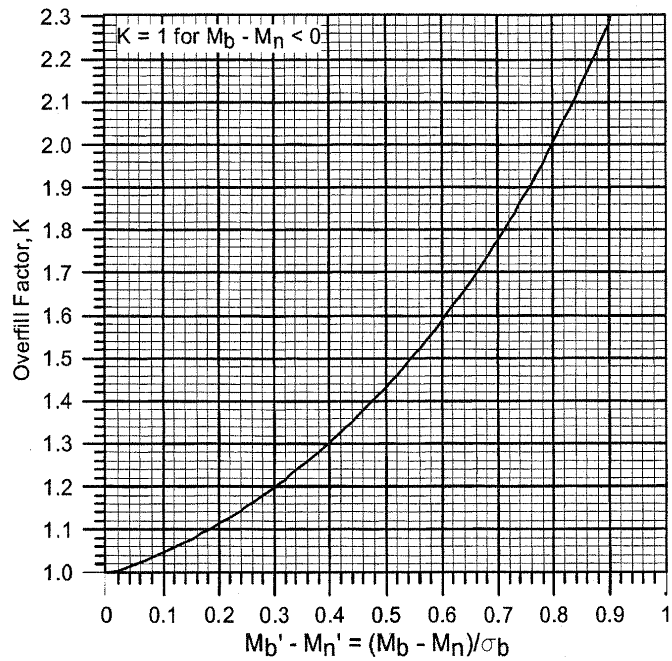

Fig. 4. Dean's overfill ratio expressed as a single curve, shown for $K<2.3$

Fig. 2, then rotating 90° clockwise to the $M_n'=0$ axis, then reading the value of K. This allows the family of overfill curves to be expressed as a single curve, K, as a function of $M_b'-M_n'$. The result is shown in Fig. 3 for a large range of K. Fig. 4 illustrates a subset of these results for a smaller range of K values of more practical engineering interest. The values of K from these graphs are identical to those in Dean (1974).

The writer did not find a numeric expression that would fit the single K curve in Figs. 3 or 4 with an error that is acceptably low for practical design application. That is, numeric approximation of the curve could be of value in pedagogical discussion; but for engineering application to actual projects, it is recommended that

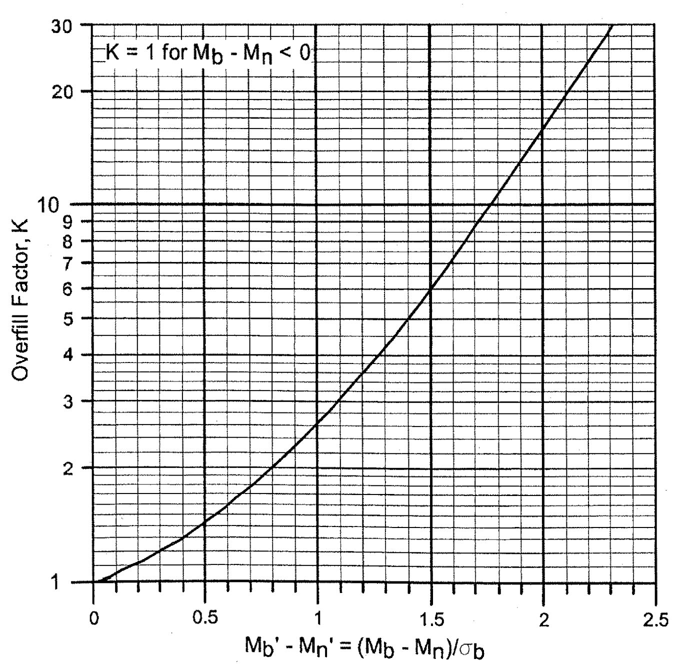

Fig. 3. Values for Dean's overfill ratio expressed as a single curve

numeric values of the Dean overfill ratio be determined from the computed curve in Fig. 3 or Fig. 4 rather than from a numerical fit to the curve.

The expression of K as a function of a single parameter—as shown in Figs. 3 and 4—readily allows a practical understanding of how Dean's overfill ratio varies with three fundamental parameters that describe the native and borrow sediment grain sizes. While these relationships are inherent in Dean's development and published description of his method, the physical significance of these relationships becomes quickly evident to the practicioner in the simple graphical results of Figs. 3 and 4.

First, Dean's method yields an overfill factor of 1.0 for any case in which the mean grain size of the borrow material is *coarser* than the native material (i.e., $M_b \leq M_n$), regardless of the sorting parameter of either the borrow or native material.

Second, for cases where the mean grain size of the borrow material is *finer* than the native, the overfill factor improves (approaches 1.0) as the borrow material's sorting parameter increases. Physically, this reflects the Dean method's fundamental premise that the fraction of borrow sediment that is *coarser* than the native sediment "makes up" for the fraction of borrow sediment that is *finer* than the native sediment. Numerically, if one strives for an "ideal" overfill factor of, say, $K=1.05$ or less from the Dean method, then the sorting parameter of the borrow material, σ_b, must be at least nine times greater than the mean grain size difference between the borrow and native material [that is, $(M_b-M_n)/\sigma_b \leq 0.11$, from Fig. 4]. Likewise, if one strives for a maximum allowable overfill factor of, say, $K=1.3$—equivalent to 30% sorting losses—then the sorting parameter of the borrow material must be at least 2.5 times greater than the mean grain size difference between the borrow and native material [that is, $(M_b-M_n)/\sigma_b \leq 0.4$].

Example Application

Figs. 5 and 6 depict sediment grain size data for two examples of borrow and native beach sediments. In each figure, the upper

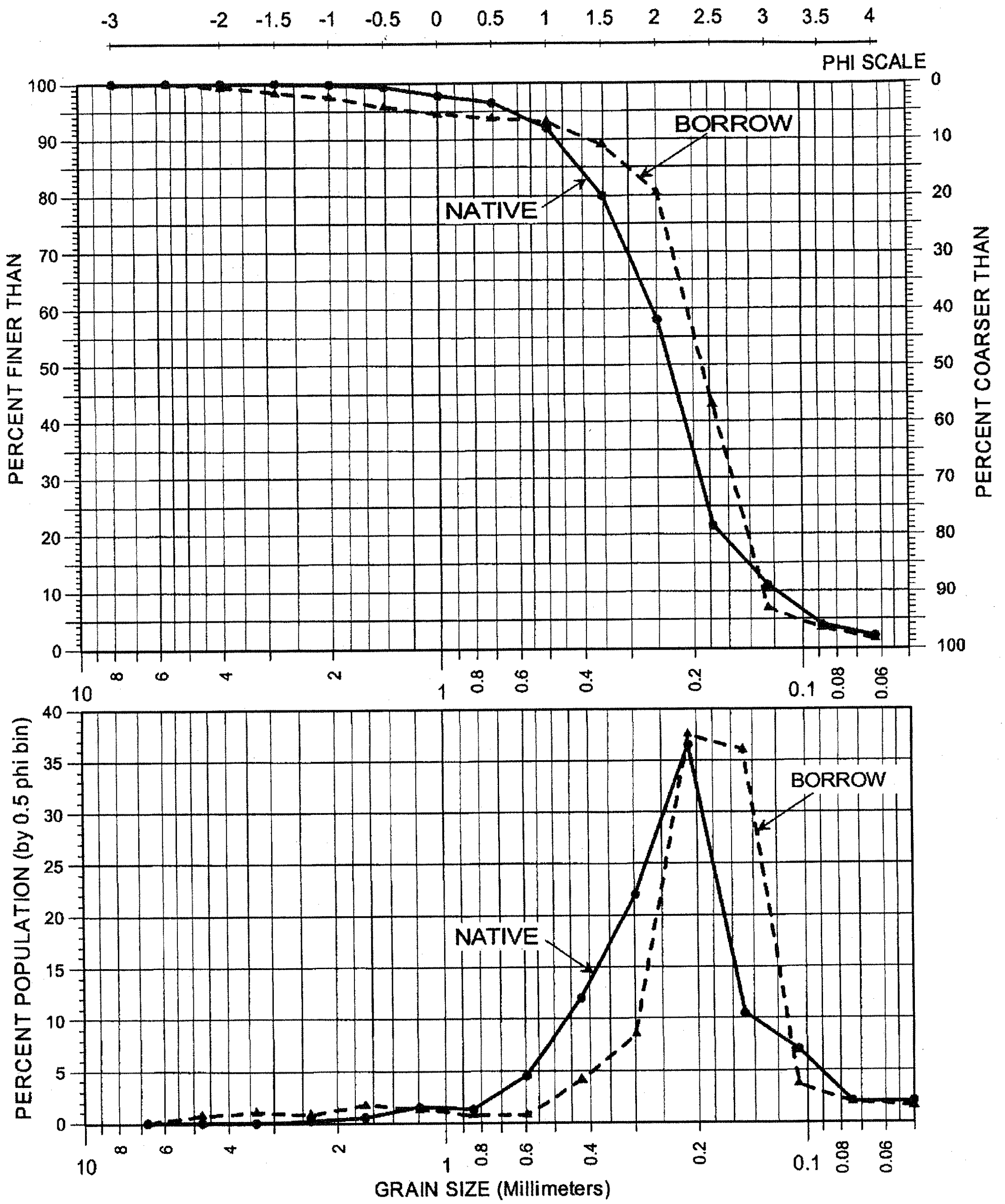

Fig. 5. Example no. 1: Grain size distribution for borrow and native sediments for which Dean overfill factor $K=1.08$ and James-Krumbein overfill factor=1.25.

graphic illustrates the cumulative grain size distribution while the lower graphic illustrates the corresponding frequency (or "population") distribution of the sediment grains. Table 1 lists the grain size statistics for each example based upon visual examination of Figs. 5 and 6 and as computed from Eqs. (4*a*)–(4*c*).

Table 1 also lists the computed parameter $(M_b-M_n)/\sigma_b$ from which Dean's overfill factor, K, is found using Fig. 3 or Fig. 4; and the table lists the parameters $(M_b-M_n)/\sigma_n$ and σ_b/σ_n from which the James-Krumbein overfill factor is found (James 1974, 1975; USACE 1984, 2001). Values for both the Dean and JK overfill factors are included in the table, in bold type.

In the two examples, the native beach sediment is identical. In example no. 1 (Fig. 5), the borrow material is finer-grained than the native sediment and is well-sorted. It exhibits a very slightly higher fraction of coarse grains but is otherwise, overall, finer than the native. The sorting parameter of the borrow material (0.96ϕ) is about 6.4 times greater than the difference between the borrow and native mean grain size (0.15ϕ). For this example, the Dean method computes an overfill factor of $K=1.08$. The corresponding overfill value from the JK method is about 1.25.

In example no. 2 (Fig. 6), the median grain size of the borrow material is the same as in example no. 1; i.e., finer than that of the native sediment. In contrast, however, the borrow material in this example is less well-sorted: it is characterized by a fine-grain population similar to the native beach plus a relatively higher fraction of coarse lag or shell fragments or the like. In this case, the mean value of the borrow material ($M_b=2.09$) is very similar to that of the native material ($M_n=2.07$), and the sorting parameter of the borrow material (1.45ϕ) is over 70 times greater than the mean size difference. The Dean method yields an overfill

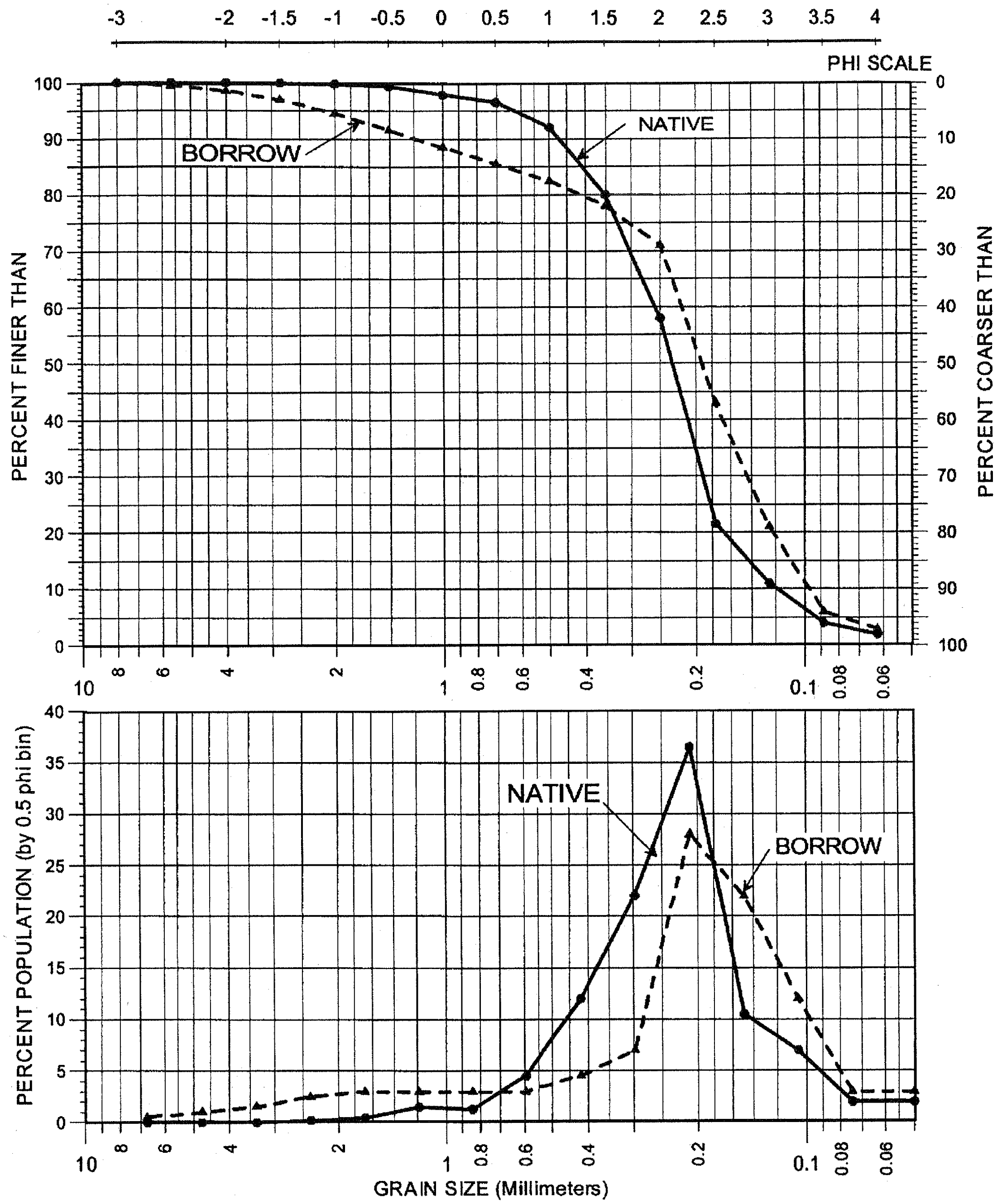

Fig. 6. Example no. 2: Grain size distribution for borrow and native sediments for which Dean overfill factor $K=1.00$ and James-Krumbein overfill factor=1.30.

factor of 1.00. The corresponding overfill factor from the JK method is about 1.30.

In both examples, each of which represent realistic cases encountered routinely in the prototype, the Dean method yields a less conservative (smaller) value of the overfill ratio relative to the James-Krumbein method. As described above, the difference is attributable to the Dean method's allowance of coarser borrow material offsetting predicted losses of finer material. The writer offers no conclusion or speculation as to which method is more accurate, and knows of no prototype data that are sufficiently well-conditioned to yield such a conclusion (given all of the other physical factors that govern beach fill performance). In practice, many experienced engineers compute the overfill ratio from both methods and weigh the significance of each method's result as a function of the general nature of the sediments, prior experience, anticipated dredging and fill-placement methods, and the level of conservatism deemed appropriate for any given project. Application of equilibrium profile concepts is often made as an additional means by which to estimate the requisite fill volume. In many practical cases where the beach is composed of well-graded sediments or significant cross-shore variations in grain size, uncertainties in specifying a single grain size distribution of the native beach material may outweigh the differences in computed overfill ratio (or fill requirements) derived from the various overfill ratio and equilibrium profile methods.

Table 1. Grain Size Statistics and Computed Overfill Ratio for Examples 1 and 2

Coarser than	Native sediment φ_n	Example 1 borrow φ_b	Example 2 borrow φ_b
5%	0.64	−0.20	−1.14
16%	1.32	1.36	0.62
50%	2.09	2.41	2.38
84%	2.79	2.88	3.25
95%	3.43	3.29	3.64
M_n	2.07	—	—
M_b	—	2.22	2.09
σ_n	0.83	—	—
σ_b	—	0.96	1.45
	$(M_b-M_n)/\sigma_b$	0.16	0.01
	K value (Dean)	**1.08**	**1.00**
	$(M_b-M_n)/\sigma_n$	0.18	0.02
	σ_b/σ_n	1.16	1.75
	K value (James-Krumbein)	**1.25**	**1.30**

Conclusion

Values of Dean's overfill ratio, K, can be estimated from the single curve given in Fig. 3 (and as enlarged in Fig. 4), using the parameter $(M_b-M_n)/\sigma_b$ where M_b and M_n=mean grain size of the borrow and native sediments [Eqs. (4*a*) and (4*b*)], respectively, and σ_b is the sorting parameter of the borrow sediment [Eq. (4*c*)], all in phi units. The curve provides for easily discriminated estimates of the overfill ratio over an unlimited range of sediment types. Values for this curve were computed directly from Dean's equations. Values for the overfill factor yielded by the single curve shown herein are identical to those depicted in the family of semilogarithmic curves given in Dean (1974). A convenient numerical means is also presented by which to convert between phi units and millimeter grain size, as given in Eqs. (3*a*) and (3*b*).

From the Dean method, an overfill factor of $K=1.05$ or less requires that the sorting parameter of the borrow material, σ_b, be at least nine times greater than the mean grain size difference between the borrow and native material, M_b-M_n, all expressed in phi units. Similarly, an overfill factor of $K=1.3$ or less requires that σ_b be at least 2.5 times M_b-M_n. The Dean method yields an overfill factor of $K=1.0$ for any case in which the mean grain size of the borrow material is coarser than that of the native material, regardless of sorting parameter.

Notation

The following symbols are used in this technical note:

d = grain size in millimeters;
K = overfill factor or overfill ratio;
M_b = mean diameter of the borrow or fill sediment (phi units);
M_n = mean diameter of the native beach sediment (phi units);
σ_b = standard deviation or sorting parameter of the borrow sediment (phi units);
σ_n = standard deviation or sorting parameter of the native beach sediment (phi units);
φ = phi-unit sediment grain size; and
φ_*, t, x = dummy variables of integration.

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